

Unscrambling the Quantum Omelette

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Based on recent theorems about quantum value-indefiniteness it is conjectured that many issues of “Born’s quantum mechanics” can be overcome by supposing that only a single pure state exists; and that the quantum evolution permutes this state.

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INGREDIENTS

The following rather “iconoclastic” recasting of quantum mechanics applies to the quantum formalism as outlined by von Neumann [1]. It will most likely survive this theory because the definitions, conventions and results presented apply to a reversible (indeed, bijective) state evolution, which amounts to permutations of elements in some state space. The title is taken from a passage of Jaynes [2], presenting the current quantum mechanical formalism as “*not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature – all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble.*”

What might be the ingredients of such a quantum omelette? First and foremost, we need to keep in mind that we are dealing with *self-perception*: no observer has a “direct, detached, objective, extrinsic” viewpoint; all observers are “embedded” in the system they observe (“Cartesian prison”) [3–5];

Second, all observations are based on *detector clicks*. Based on these clicks, and through *projections and conventions* of our mind we reconstruct what we consider the physical universe. Any inductive (re-)construction of a representation of a universe entirely from “physical signals” and, in particular, from detector clicks, is a subtle epistemic and physical task [6, 7] involving explicit and implicit conventions and assumptions. As we do not possess any direct access to the system other than these clicks we have to be careful in ascribing physical properties and existence to anything [8]. Indeed, it must be expected that we are deceived by our preconceptions, implicit conventions, and subjective expectations and projections. Jaynes called this the “Mind Projection Fallacy” [2, 9], pointing out that “*we are all under an ego-driven temptation to project our private thoughts out onto the real world, by supposing that the creations of one’s own imagination are real properties of Nature, or that one’s own ignorance signifies some kind of indecision on the part of Nature.*” I believe that this “over-interpretation of empirical data,” in particular, of detector clicks, is at the heart of many misconceptions about quantized sys-

tems.

Let us, as a starter, mention some quantum examples of the Mind Projection Fallacy. First, consider the inclinations [10] yielding claims [11] of absolute, irreducible indeterminism and randomness, demanding the “*ex nihilo* emergence of single bits (of information).” In this orthodox line of thought, the apparent lack of prediction and control is not merely “means-relative” [12]; that is, the possibility of mere epistemic ignorance, originating from the limited capacities of intrinsic observers, resulting in “pragmatic” propositions that are true “for all practical purposes” [13] (FAPP) but strictly false, is denied.

Rigorously speaking, any believe in (in-)determinism is *provably improvable* because, by reduction to recursion theoretic unknowables (e.g., the halting problem or the rule inference problem [14–18]), randomness as well as determinism turn out to be undecidable. That is, one may still be “inclined to believe in (in-)determinism” [10], and this believe might serve as a good, pragmatic working hypothesis for various tasks; alas, strictly speaking, any such “evidence” is no more compelling than, say, the belief in Santa Claus.

An algorithmic proof can be sketched as follows: suppose Bob presents Alice a black box, thereby claiming that the box contains an oracle for indeterminism, or even absolute randomness. Alice’s challenge is to “verify” that this is correct. As it turns out, Alice’s verification task is impossible if she is bound by intrinsic algorithmic means, because every time Alice has made up her mind that no algorithm from a particular finite set of algorithms is generating the output of the box, by diagonalization Bob can construct a “faker box algorithm” which yields a different output than Alice’s finite set of algorithms; thereby giving Alice the wrong illusion of randomness. With finite physical means the limit of “all (i.e., a countable infinity of) algorithms” is impossible to attain. But even for a finite number of algorithms, their output behavior is FAPP impossible to predict, since the halting time of a program of fixed length is of the order of the Busy Beaver function of that length, and therefore grows faster than any computable function thereof [19].

Suppose, on the other hand, that Bob claims that the

box behaves deterministically. In this case, Alice can be deceived as well; because whenever she claims to know such an algorithm, by diagonalization Bob can fabricate another “faker algorithm” which behaves exactly as Alice’s algorithm until she mentions her claim, and then behaves differently. In that way, Alice will never be able to prove determinism.

Of course, the obvious “solution” would be to allow Alice to “screw open Bob’s box” and see whether contained in it there is any “paper-and-pencil Turing type machinery;” alas this is not allowed in the intrinsic epistemology.

Other fallacies involve so-called “*experimental proofs of the Kochen-Specker (KS) theorem*” – because “how can you measure a [proof by] contradiction?” [20]; as well as “*experimental proofs of contextuality*” – what is actually measured are violations of Boole-Bell type inequalities *via* successive measurements of counterfactual, complementary observables that are not co-measurable [21]. Although contextuality might be *sufficient* to render any experimental records (even beyond quantum correlations [22]), these experiments fall short of any strict test of the *necessity* of contextuality.

Still another fallacy is the assumption of the *physical (co-)existence of counterfactuals* (Specker’s “Infuturabilien” referring to scholastic debates); that is, hypothetical observables that one could have, but did not measure; instead some different, complementary, observable has been measured. We shall come back to this issue later. Finally let me mention the fallacy of supposing that there is some *space-time theater* in which events occur; rather than the “operationalization” of space-time *via* events [23, 24].

ONTOLOGICAL SINGLE PURE STATE CONJECTURE

So, in view of these epistemic limitations and pitfalls, how might one be able to “unscramble” the quantum omelette? In what follows, the KS and related theorems will be used as a guiding principle. But first, we need to clarify what constitutes a pure quantum state.

Definition 1 (State). Informally, we shall assume that a *pure state* is characterized by the *maximal information* encodable into a physical system. This amounts to a generalized beam splitter configuration [25] with an array of detectors; of which only one clicks, the others remain silent. Formally, a pure quantum state can be represented by a two-valued measure either (i) on an *orthonormal basis*; or (ii) on the spectral decomposition of a *maximal operator*, from which all commuting orthogonal projectors corresponding to (i) can be functionally derived (they occur in the spectrum); or (iii) on a *context, subalgebra* or *block*; or (iv) on the constituents of

a *unitary transformation* “encoding” the basis states (i) by, say, arranging the coordinates of the basis as either rows or columns in a matrix representation.

The (strong) KS theorem is usually proved by taking a finite subset of interconnected (the dimension of the vector space must be three or higher for interconnectivity) contexts (or any similar encoding thereof, such as maximal observables, orthogonal bases, or unitary operators), and by demonstrating that no two-valued measure (interpretable as classical truth assignment) exists on those structures of observables if non-contextuality is required – meaning that the measure is independent of the context. In a classical (non-contextual) sense, “somewhere” in these finite constructions any attempt to overlay a two-valued measure – that is, any enumeration of truth assignments regarding the propositions about outcomes of conceivable measurements – must break down due to inconsistencies. This also occurs, at least for some members of an ensemble, in Boole-Bell-type configurations [26]. Other weak forms of the KS theorem allow two-valued measures, alas they may be too scarce to, for instance, be able to separate all observables; and to allow a homeomorphic embedding into Boolean algebras.

Pitowsky’s *logical indeterminacy principle* [27, 28] as well as the Abbott-Calude-Conder-Svozil (ACCS) theorem [29] can, in a particular sense, be considered an “improved” version of the KS theorem which certifies “breakdown of (non-contextual) value definiteness” for observable $|\mathbf{b}\rangle\langle\mathbf{b}|$ (associated with the vector $|\mathbf{b}\rangle$); from now on, the vector and its associated projector will be used synonymously, if the quantum is prepared in a particular state such that the observable $|\mathbf{c}\rangle$, which must not be orthogonal and collinear to $|\mathbf{b}\rangle$, occurs with certainty. More formally, by considering some construction of interconnected contexts it turns out that both possible value assignments $v(|\mathbf{b}\rangle) = 0$ as well as $v(|\mathbf{b}\rangle) = 1$ are inconsistent with the value assignment $v(|\mathbf{c}\rangle) = 1$ for any non-orthogonal and non-collinear $|\mathbf{b}\rangle$ (in the case of ACCS, for proof technical reasons, $\sqrt{5/14} \leq |\langle\mathbf{c}|\mathbf{b}\rangle| \leq 3/\sqrt{14}$).

By excluding certain *star-shaped configurations of contexts* characterized by an arbitrary number of orthogonal bases with one common element (cf. Fig. 1), it is possible to extend Pitowsky’s logical indeterminacy principle and the ACCS theorem to the remaining “observables.”

For the sake of demonstration, consider a configuration of three vectors $|\mathbf{a}\rangle \perp |\mathbf{c}\rangle \not\perp |\mathbf{b}\rangle$, and a two-valued state $v(|\mathbf{c}\rangle) = 1$. Note that $|\mathbf{a}\rangle$ lies on the plane (through the origin) orthogonal to $|\mathbf{c}\rangle$, whereas $|\mathbf{b}\rangle$ lies outside of this orthogonal plane. In terms of Greechie orthogonality diagrams [30], $|\mathbf{a}\rangle$ as well as $|\mathbf{c}\rangle$ are contained in a star-shaped configuration of contexts characterized by the rays perpendicular to some “true” $|\mathbf{c}\rangle$ with $v(|\mathbf{c}\rangle) = 1$; whereas $|\mathbf{b}\rangle$ lies outside of “ $|\mathbf{c}\rangle$ ’s star.” For any such observable corresponding to $|\mathbf{b}\rangle$ there is no consistent two-value state assignment whatsoever.

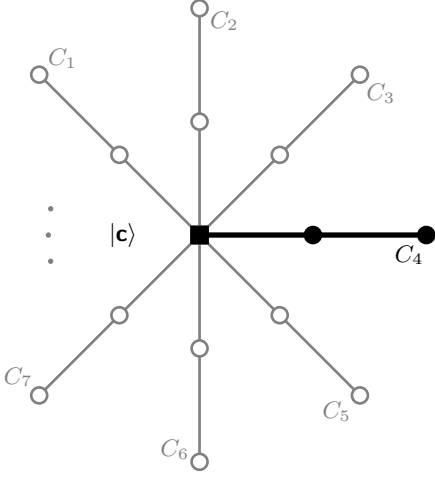


FIG. 1. (Color online) Greechie orthogonality diagram of a star-shaped configuration, representing a common detector observable $|c\rangle\langle c|$ with an overlaid two-valued assignment reflecting $v(|c\rangle) = 1$. It is assumed that the system is prepared in state C_4 , depicted by a block colored in thick filled black; all the other (continuity of) contexts are “phantom contexts” colored in gray. (Compare also Ref. [29, Fig. 2].)

That is, if $|a\rangle$ is orthogonal to $|c\rangle$ the value assignment $v(|a\rangle) = 0$ follows from $v(|c\rangle) = 1$; but it is inconsistent with either $v(|b\rangle) = 0$ or $v(|b\rangle) = 1$ for all $|b\rangle$ non-orthogonal and non-collinear to $|c\rangle$. This is a direct consequence of Pitowsky’s logical indeterminacy principle, which, given $v(|c\rangle) = 1$, does not allow any two-valued state $v(|b\rangle) = 0$ or $v(|b\rangle) = 1$.

For the configuration $|a\rangle \not\perp |c\rangle \not\perp |b\rangle$, both $|a\rangle$ as well as $|b\rangle$ lie outside of “ $|c\rangle$ ’s star,” and are thus value indefinite. On the other hand, if we assume $|a\rangle \perp |c\rangle \perp |b\rangle$ – that is, both $|a\rangle$ as well as $|b\rangle$ are orthogonal to $|c\rangle$ (and thus “in $|c\rangle$ ’s star”) – $v(|a\rangle) = v(|b\rangle) = 0$, even if they are non-orthogonal. Hence, given $v(|c\rangle) = 1$, relative to the KS assumptions, the only consistent assignments may be made “inside $|c\rangle$ ’s star.” “Outside of $|c\rangle$ ’s star” all “observables” are value indefinite (relative to the KS assumptions, including non-contextuality).

How can one utilize these findings? One immediate possibility is the construction of a *quantum random number generator* “certified by quantum value indefiniteness:” prepare $|c\rangle$, measure $|b\rangle\langle b|$ [29].

Another, intuitive speculation based on the very limited value-definiteness allowed by the KS assumptions (including non-contextuality) suggests a very simple principle. While extensions of the logical indeterminacy principle and the ACCS theorem will never be able to go beyond value indefiniteness of all but a “star-shaped” configuration of contexts depicted in Fig. 1, I suggest to “get rid” of even star-shaped configurations by denying the physical co-existence of all but one context – the one in which the quantum has been “prepared” – prior to

measurement.

Conjecture 1 (Ontological single pure state conjecture). “A quantized system is in a state corresponding to a two-valued measure on a single definite context (orthonormal basis, block, maximal observable, unitary operator).” In terms of observables, this translates into “ontologically there does not exist any observable beyond the observables representing a single definite context.”

The ontological single pure state conjecture claims that a single quantum state is a *complete* theoretical representation of a physical system. Thereby it *abandons omniscience and omniscience*: it states that all other (even hypothetically and consistently “value definite” yet counterfactual) observables different from the observables associated with the unique state, and possibly ascribed to such a system, are not value definite at all.

One should not be “tricked” into believing that such value indefinite observables are “measurable” just because their alleged “measurement” yields outcomes; that is, clicks in detectors that one identifies with (pre-existing) values. These *outcomes cannot reflect any value definite property of the object prior to measurement* because, according to the single pure state conjecture, such a value definite property simply does not exist. Rather the detector clicks associated with the “measurement” might be a very complex consequence of “*the complete disposition of the apparatus*” [31], as well as of the object, combined. In contradistinction, orthodox quantum mechanics treats *all potentially conceivable* observables on an *equal footing*.

We shall also introduce two other concepts: a *phantom context*, and *context translation*: Any context that is not the single context/state (in which the system is prepared) is a *phantom context*. And any mismatch between the preparation and the measurement may result in the *translation* of the original information encoded in a quantum system [32] into the answer requested, whereby noise is introduced by the many degrees of freedom of a suitable “quasi-classical” measurement apparatus.

For the sake of demonstration of these concepts, consider the rule that, under the KS assumptions (including non-contextuality), for Specker’s “bug” configuration (Pitowsky’s “cat’s cradle” graph) of contexts as depicted in Fig. 2, if a classical system is prepared in a two-valued state on the context C_1 , with $v(|a\rangle) = 1$ (i.e. the detector corresponding to observable $|a\rangle$ clicks), and with $v(|c\rangle) = v(|d\rangle) = 0$ (i.e. the detectors corresponding to observables $|c\rangle$ and $|d\rangle$ do not click), then $v(|b\rangle) = 0$; that is, a detector corresponding to observable $|b\rangle$ will not click. [A rather simple proof by contradiction (wrongly) assumes that $v(|a\rangle) = 1$ as well as $v(|b\rangle) = 1$ can co-exist consistently, thereby leading to a complete contradiction, since in this case the value assignment of both link observables for C_3/C_5 as well as C_4/C_5 have to be 1, alas these link observables belong to the same block

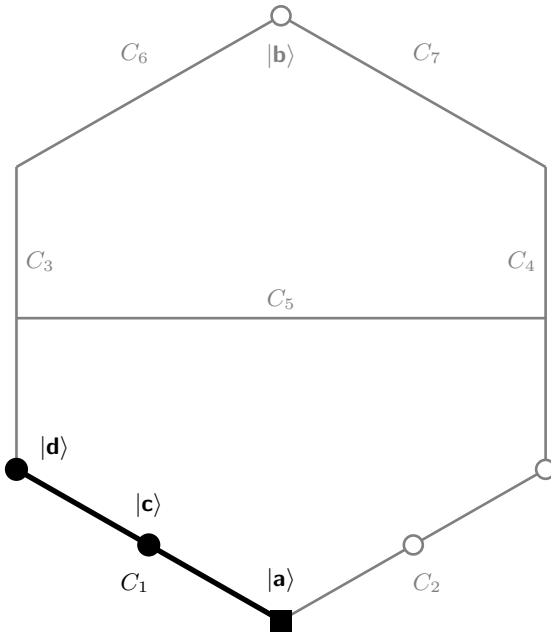


FIG. 2. (Color online) “Bug-type” [34] Greechie orthogonality diagram with an overlaid two-valued assignment reflecting “ $v(|\mathbf{a}\rangle) = 1$ implies $v(|\mathbf{b}\rangle) = 0$.” This configuration is part of the original proof of the KS theorem [35, Γ_1]. For concrete coordinatizations, see, for instance, the original paper by Kochen and Specker, as well as Ref. [29, 33]. It is assumed that the system is prepared in state C_1 , depicted by a block colored in thick filled black; all the other six remaining contexts C_2 – C_7 are “phantom contexts” colored in gray.

C_5 .] That quantum mechanics contradicts this prediction “if $v(|\mathbf{a}\rangle) = 1$ then $v(|\mathbf{b}\rangle) = 0$ ” is an immediate consequence of the fact that, because $|\mathbf{a}\rangle$ and $|\mathbf{b}\rangle$ are not in the same block, $|\mathbf{a}\rangle$ cannot be orthogonal to $|\mathbf{b}\rangle$, and hence $\langle \mathbf{a} | \mathbf{b} \rangle \neq 0$, implying a non-vanishing probability $|\langle \mathbf{a} | \mathbf{b} \rangle|^2 \geq 0$. For a concrete though not unique parametrization of the “bug” configuration, see Fig. 4.2 in Ref. [33], in which preparation of $|\mathbf{a}\rangle \equiv (1/\sqrt{3})(\sqrt{2}, 1, 0)$ and measurement of $|\mathbf{b}\rangle \equiv (1/\sqrt{3})(\sqrt{2}, -1, 0)$ implies a probability of observing $|\mathbf{b}\rangle$, given $|\mathbf{a}\rangle$ of $|(1/\sqrt{3})(\sqrt{2}, 1, 0) \cdot (1/\sqrt{3})(\sqrt{2}, -1, 0)|^2 = 1/9$ (and not zero, as predicted from classical non-contextuality).

However, since according to the single pure state conjecture only C_1 exists, any argument based on the simultaneous co-existence of the counterfactual phantom contexts C_2 – C_7 , and, in particular, the assumption of a property associated with the counterfactual observable $|\mathbf{b}\rangle\langle\mathbf{b}|$, is inadequate for quantized systems.

PERSISTENT ISSUES

Do measurements exist?

Everett [36] and Wigner [37] observed that, if a unitary (bijective, one-to-one, reversible, Laplacian-type deterministic) quantum evolution were universally valid, then any distinction or cut between the observer and the measurement apparatus on the one side, and the quantum “object” on the other side, is not absolute or ontic, but epistemic, means-relative, subjective and conventional.

Because, suppose that one has defined a cut or difference between some quantum and a “quasi-classical” measurement device, one could, at least in principle and if the unitary quantum evolution is universally valid, “draw a larger perimeter.” This “enlargement” could contain the entire previous combination, *including* the quantum, the cut, and the measurement device. If the quantum laws are universally valid, such a quantized system should also undergo a unitary quantum evolution. And thus, if quantum mechanics is universally valid, and if it is governed by unitary, reversible, one-to-one evolution, how could irreversibility possibly “emerge” from reversibility? FAPP, due to the limitations of the experimenter’s capacities irreversibility may be means-relative; alas, strictly speaking, it decays into “thin air.”

Because suppose (wrongly) a hypothetical many-to-one function $h(x) = h(y)$ for $x \neq y$ exists which would somehow ‘emerge’ from injective functions. Any such function would have to originate from the domain of one-to-one functions such that, for all functions f of this class, $x \neq y$ implies $f(x) \neq f(y)$ – or, equivalently, the contrapositive statement (provable by comparison of truth tables) $f(x) = f(y)$ implies $x = y$, a clear contradiction with the assumption.

Indeed, by *Cayley’s theorem* the unitary transformations on some Hilbert space \mathfrak{H} form a particular permutation group consisting of those permutations preserving the inner product. This is a subgroup of the *symmetric group* of all permutations on \mathfrak{H} . So, strictly speaking, any quantum mechanical state evolution amounts to permuting the state, and therefore leaves no room for “measurement.”

Quantum jellification

Alas, as Schrödinger pointed out, without measurement every quantum physicist should be troubled that, due to the coherent superposition resulting from the co-existence of classically mutually exclusive alternatives, their “surroundings rapidly turning into a quagmire, a sort of a featureless jelly or plasma, all contours becoming blurred, we ourselves probably becoming jelly fish” [38].

The single pure state conjecture and the context translation principle would resolve this conundrum by main-

taining that there is only one state “perceived” from many epistemic perspectives [39]; some of them causing noise which FAPP appears irreducible random to intrinsic observers. In that sense, the measurement conundrum, with all its variants – Schrödinger’s cat of jellyfish metaphors, as well as the Everett-Wigner critique – is “FAPP-resolved by means-relativity.”

Analogues in classical statistical mechanics

Just as Newtonian physics and electromagnetism appear to be reversible, the quantum measurement conundrum is characterized by the reversibility of the unitary quantum evolution. In this respect, the (ir-)reversibility of quantum measurements bears some resemblance to statistical mechanics: take, for example, *Loschmidt’s reversibility paradox* – that, for large isolated systems with reversible laws of motion, one should never observe irreversibility, and thus a decrease in entropy; or *Zermelo’s recurrence objection* – that, as an isolated system will infinitely often approach its initial state, its entropy will infinitely often approach the initial entropy and thus cannot constantly increase; or the challenge posed by the *Loschmidt-Maxwell demon* [40]. And just as in statistical mechanics, irreversibility appears to be means-relative [12] and FAPP, yet cannot strictly be true.

The epistemic or ontic (non-)existence of mixed states

From a purely formal point of view, it is impossible to obtain a mixed state from a pure one. Because again, any unitary operation amounts to a mere basis transformation or permutation, and this cannot give rise to any increase in stochasticity or “ignorance.” Since the generation of “ontologically mixed states” from pure ones would require a many-to-one functional mapping, we conclude that, just as irreversible measurements, genuine “ontological mixed states” originating from pure states cannot exist. Therefore, any ontological mixed state has to be either carried through from previously existing mixed states (if they exist), or be FAPP perceived as means-relative. I would like to challenge anyone with doubts to come up with a concrete experiment that would “produce” a mixed state from a pure one by purely quantum mechanical “unitary” means.

SUMMARY

In summary I hold these conjectures to be true: a quantum state characterized by the maximal information encoded into a physical system must formally be represented by some orthonormal basis and a two-valued

measure thereon, or anything encoding it, such as a maximal operator. At any given moment, a quantized system is in a unique, single such state. All other contexts are phantom contexts, which have no meaning because they are non-operational at best, and in general misleading. Randomness does not come about *ex nihilo* but by *context translation*, whereby the many degrees of freedom of the measurement apparatus contribute to yield means-relative, FAPP random outcomes. Finally, also mixed states are means-relative and exist FAPP, but not strictly.

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